

# On the Aharonov-Casher scattering in a CPT-odd Lorentz-violating background

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## Abstract

The effects of a Lorentz symmetry violating background vector on the Aharonov-Casher scattering is considered. By using the self-adjoint extension method we found that there is an additional scattering for any value of the self-adjoint extension parameter and non-zero energy bound states for negative values of this parameter. Expressions for the energy bound-states, phase-shift and the scattering matrix are explicitly determined in terms of the self-adjoint extension parameter. The expression obtained for the scattering amplitude reveals that the helicity is not conserved in this scenario.

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## I. INTRODUCTION

The standard model extension (SME) proposed by Colladay and Kostelecký [1–3] (cf. also [4–6]) has been an usual framework for investigating signals of Lorentz violation in physical systems and has inspired a great deal of investigations in this theme in recent years. The interest in this issue appears in the different contexts, such as field theory [7–23], aspects on the gauge sector of the SME [24–30], quantum electrodynamics [31–36], and astrophysics [37–39]. These many contributions have elucidated effects induced by Lorentz violation and served to set up stringent upper bounds on the Lorentz-violating (LV) coefficients [40]. The physical properties of the physical systems can be accessed by including in all sectors of the minimal standard model LV terms. In the fermion sector, for example, this violation is implemented by introducing two CPT-odd terms,  $V_\mu \bar{\psi} \gamma^\mu \psi$ ,  $W_\mu \bar{\psi} \gamma_5 \gamma^\mu \psi$ , where  $V_\mu$ ,  $W_\mu$  are the LV backgrounds. The LV terms are generated as vacuum expectation values of tensors defined in a high energy scale. The SME has also been used as a framework to propose Lorentz violation [41, 42] and CPT [43–48] probing experiments, which have amounted to the imposition of stringent bounds on the LV coefficients. By carefully analyzing the sectors of the SME some authors have specialized in introducing news nonminimal couplings between fermionic and gauge fields in the context of the Dirac equation. In Ref. [49], for example, a LV and CPT-odd nonminimal coupling between fermions and the gauge field was proposed in the form

$$D_\mu = \partial_\mu + ieA_\mu + i\frac{g}{2}\epsilon_{\mu\lambda\alpha\beta}(k_{AF})^\lambda F^{\alpha\beta}, \quad (1)$$

in the context of the Dirac equation,

$$(i\gamma^\mu D_\mu - M)\Psi = 0, \quad (2)$$

where  $\Psi$  is the fermion spinor,  $(k_{AF})^\mu = (V_0, \mathbf{V})$  is the Carroll-Field-Jackiw four-vector,  $g$  is a constant that measures the nonminimal coupling magnitude, and  $F^{\mu\nu}$  is the electromagnetic field tensor, with

$$F^{0i} = -F^{i0} = E^i, \quad F^{ij} = -F^{ji} = \epsilon^{ijk} B_k. \quad (3)$$

This suggests that the LV background, intervening in spacetime, may correct or generate some new properties to the particles. The analysis of the nonrelativistic limit of Eq. (2) reveals that the nonminimal coupling of the background with the electromagnetic fields generates a magnetic dipole moment  $g\mathbf{V}$  even for non-charged particles [49], yielding an Aharonov-Casher (AC) phase

[50] for its wave function. The nonminimal coupling in Eq. (1) has been applied to several physical systems in relativistic quantum mechanics [51–58]. Recently, a new CPT-even and LV dimension-five nonminimal coupling between fermionic and gauge fields, involving the CTP-even and Lorentz-violating gauge tensor of the SME was proposed in Ref. [59]. This new nonminimal coupling was identified by

$$D_\mu = \partial_\mu + ieA_\mu + \frac{\lambda}{2} (K_F)_{\mu\nu\alpha\beta} \gamma^\nu F^{\alpha\beta}, \quad (4)$$

with  $(K_F)_{\mu\nu\alpha\beta}$  being the tensor ruling the Lorentz violation in the CPT-even electrodynamics of the SME. This nonminimal coupling modifies the Dirac equation, whose nonrelativistic regime is governed by a Hamiltonian which induces new effects, such as an electric-Zeeman-like spectrum splitting and an anomalous-like contribution to the electron magnetic moment, among others.

In this paper, we specialize to the nonminimal coupling in Eq. (1). The aim is to study the effects of this LV background in the scattering process of a spin-1/2 neutral particle with magnetic dipole moment  $g\mathbf{V}$  in the presence of a electric field of an infinitely long, infinitesimally thin line of charge.

The work is outlined in the following way: In Section II we derive the Schrödinger-Pauli equation in order to study the physical implications of the LV background on the spin-1/2 AC scattering problem. The Section III is devoted to the study of the LV Hamiltonian via the self-adjoint extension technique and are presented some important properties of the LV wave function. In Section IV are addressed the scattering and bound-state problems within the framework of the LV Schrödinger-Pauli equation. Expressions for the energy bound-states, phase-shift and the scattering matrix are computed and all them are explicitly described in terms of the physical condition of the problem and, as it was expected, the self-adjoint extension parameter is also expressed in terms of the physical parameters. At the end, we make a detailed analysis of the helicity conservation's problem in the present framework. In Section V we give our conclusions and remarks.

## II. THE EQUATION OF MOTION

In this section, we derive the equation of motion that governs the dynamics of a spin-1/2 neutral particle in a radial electric field and a LV background. We start with the (2+1)-dimensional Dirac equation, which follows from the decoupling of (3+1)-dimensional Dirac equation for the specialized case where  $\partial_3 = 0$ , into two uncoupled two-component equations, such as implemented in Refs. [60–62]. Since we are interested only in the effects of the LV background, we can consider only the

sector generating the AC effect in Eq. (1). In this case, the planar Dirac equation ( $\hbar = c = 1$ ) is

$$(\beta\gamma \cdot \mathbf{\Pi} + \beta M)\Psi = \bar{E}\Psi, \quad (5)$$

where  $\Psi$  is a two-component spinor,  $\mathbf{\Pi} = \mathbf{p} - gs(\mathbf{V} \times \mathbf{E})$  is the generalized momentum, and  $s$  is twice the spin value, with  $s = +1$  for spin “up” and  $s = -1$  for spin “down”. The  $\gamma$ -matrices in  $(2+1)$  are given in terms of the Pauli matrices

$$\beta = \gamma_0 = \sigma_3, \quad \gamma_1 = i\sigma_2, \quad \gamma_2 = -is\sigma_1. \quad (6)$$

The field configuration (in cylindrical coordinates) is chosen to be

$$\mathbf{E} = \frac{\lambda}{2\pi\epsilon_0} \frac{\hat{\mathbf{r}}}{r}, \quad \nabla \cdot \mathbf{E} = \frac{\lambda}{2\pi\epsilon_0} \frac{\delta(r)}{r}, \quad (k_{AF})^\mu = (0, 0, 0, V), \quad (7)$$

where  $\mathbf{E}$ , is the electric field generated by an infinite charge filament and  $\lambda$  is the charge density along the  $z$ -axis. The second order equation implied by Eq. (5) is obtained by applying the matrix operator  $[M + \beta\bar{E} - \gamma \cdot \mathbf{\Pi}] \beta$ . After this application, one finds

$$\begin{aligned} (\bar{E}^2 - M^2)\Psi &= -(\gamma \cdot \mathbf{\Pi})(\gamma \cdot \mathbf{\Pi})\Psi \\ &= \{\mathbf{\Pi}^2 + g\sigma \cdot [\nabla \times (\mathbf{V} \times \mathbf{E})]\}\Psi. \end{aligned} \quad (8)$$

By accessing the nonrelativistic limit,  $E = M + \bar{E}$ ,  $M \gg \bar{E}$ , we obtain the Schrödinger-Pauli equation

$$\hat{H}\Psi = E\Psi, \quad (9)$$

with

$$\hat{H} = \frac{1}{2M} [\mathbf{p} - gs(\mathbf{V} \times \mathbf{E})]^2 + \frac{1}{2M} g\sigma \cdot [\nabla \times (\mathbf{V} \times \mathbf{E})], \quad (10)$$

the Hamiltonian operator. Using (7), the Hamiltonian (10) becomes

$$\hat{H} = \frac{1}{2M} \left[ \hat{H}_0 + \alpha\sigma_z \frac{\delta(r)}{r} \right], \quad (11)$$

with

$$\hat{H}_0 = \left( \frac{1}{i} \nabla - \alpha s \frac{\hat{\mathbf{r}}}{r} \right)^2, \quad (12)$$

and

$$\alpha = \frac{gV\lambda}{2\pi\epsilon_0}, \quad (13)$$

is the coupling constant of the  $\delta(r)/r$  potential.

The Hamiltonian in Eq. (11) governs the quantum dynamics of a spin-1/2 neutral particle with a radial electric field, i.e., a spin-1/2 AC problem, with  $g\mathbf{V}$  playing the role of a nontrivial magnetic dipole moment, in contrast with the usual AC problem where the magnetic dipole moment is  $\boldsymbol{\mu} = \mu\sigma_z\hat{\mathbf{z}}$  [63]. Also, note the presence of a  $\delta$  function which is singular at the origin in Eq. (11). This makes the problem more complicated to be solved. Such kind of point interaction potential can then be addressed by the self-adjoint extension approach [64, 65], which will be used for studying the scattering and bound state scenarios.

### III. SELF-ADJOINT EXTENSION ANALYSIS

An operator  $\mathcal{O}$ , with domain  $\mathcal{D}(\mathcal{O})$ , is said to be self-adjoint if and only if  $\mathcal{D}(\mathcal{O}^\dagger) = \mathcal{D}(\mathcal{O})$  and  $\mathcal{O}^\dagger = \mathcal{O}$ . In order to determine all self-adjoint extensions of (12), making use of the underlying rotational symmetry expressed by the fact that  $[\hat{H}, \hat{J}_z] = 0$ , where  $\hat{J}_z = -i\partial/\partial\varphi + \sigma_z/2$  is the total angular momentum operator in the  $z$ -direction, we decompose the Hilbert space  $\mathfrak{H} = L^2(\mathbb{R}^2)$  with respect to the total angular momentum  $\mathfrak{H} = \mathfrak{H}_r \otimes \mathfrak{H}_\varphi$ , where  $\mathfrak{H}_r = L^2(\mathbb{R}^+, r dr)$  and  $\mathfrak{H}_\varphi = L^2(\mathcal{S}^1, d\varphi)$ , with  $\mathcal{S}^1$  denoting the unit sphere in  $\mathbb{R}^2$ . So, it is possible to express the eigenfunctions of the two dimensional Hamiltonian in terms of the eigenfunctions of  $\hat{J}_z$

$$\Psi(r, \varphi) = \begin{pmatrix} \psi_m(r)e^{i(m_j-1/2)\varphi} \\ \chi_m(r)e^{i(m_j+1/2)\varphi} \end{pmatrix}, \quad (14)$$

with  $m_j = m + 1/2 = \pm 1/2, \pm 3/2, \dots$ , and  $m \in \mathbb{Z}$ . By inserting Eq. (14) into Eq. (9) the Schrödinger-Pauli equation for  $\psi_m(r)$  is found to be ( $k^2 = 2ME$ )

$$H\psi_m(r) = k^2\psi_m(r), \quad (15)$$

where

$$H = H_0 + \alpha \frac{\delta(r)}{r}, \quad (16)$$

and

$$H_0 = -\frac{d^2}{dr^2} - \frac{1}{r} \frac{d}{dr} + \frac{(m - \alpha s)^2}{r^2}. \quad (17)$$

The self-adjoint extension approach consists, essentially, in extending the domain  $\mathcal{D}(H_0)$  to match  $\mathcal{D}(H_0^\dagger)$  and therefore turning  $H_0$  into a self-adjoint operator. To do so, we must find the deficiency subspaces,  $N_\pm$ , with dimensions  $n_\pm$ , which are called deficiency indices of  $H_0$  [66]. A

necessary and sufficient condition for  $H_0$  being essentially self-adjoint is that  $n_+ = n_- = 0$ . On the other hand, if  $n_+ = n_- \geq 1$ , then  $H_0$  has an infinite number of self-adjoint extensions parametrized by unitary operators  $U : N_+ \rightarrow N_-$ . In order to find the deficiency subspaces of  $H_0$  in  $\mathfrak{H}_r$ , we must solve the eigenvalue equation

$$H_0^\dagger \psi_\pm = \pm i k_0^2 \psi_\pm, \quad (18)$$

where  $k_0^2 \in \mathbb{R}$  was introduced for dimensional reasons. Since  $H_0^\dagger = H_0$ , the solutions of Eq. (18) which vanishes at the infinite are the Hankel functions (up to a constant)

$$\psi_\pm = H_{|m-\alpha s|}^{(1)}(\sqrt{\mp i} k_0 r), \quad (19)$$

with  $\text{Im}\sqrt{\pm i} > 0$ . The dimension of such deficiency subspace is thus  $(n_+, n_-) = (1, 1)$ . According to the von Neumann-Krein theory, all self-adjoint extensions  $H_{\theta,0}$  of  $H_0$  are given by the one-parameter family

$$\mathcal{D}(H_{\theta,0}) = \mathcal{D}(H_0^\dagger) = \mathcal{D}(H_0) \oplus N_+ \oplus N_-. \quad (20)$$

Thus,  $\mathcal{D}(H_{\theta,0})$  in  $\mathfrak{H}_r$  is given by the set of functions [66]

$$\psi_\theta(r) = \psi_m(r) + c \left[ H_{|m-\alpha s|}^{(1)}(\sqrt{-i} k_0 r) + e^{i\theta} H_{|m-\alpha s|}^{(1)}(\sqrt{i} k_0 r) \right], \quad (21)$$

where  $\psi_m(r)$ , with  $\psi_m(0) = \dot{\psi}_m(0) = 0$  ( $\dot{\psi} \equiv d\psi/dr$ ), is the regular wave function,  $c \in \mathbb{C}$  and the number  $\theta \in [0, 2\pi)$  represents a choice for the boundary condition.

All the self-adjoint extensions  $H_{0,\lambda_m}$  of  $\tilde{H}_0$  are parametrized by the boundary condition at the origin [64, 65]

$$\psi^{(0)} = \lambda_m \psi^{(1)}, \quad (22)$$

with

$$\begin{aligned} \psi^{(0)} &= \lim_{r \rightarrow 0^+} r^{|m-\alpha s|} \psi(r), \\ \psi^{(1)} &= \lim_{r \rightarrow 0^+} \frac{1}{r^{|m-\alpha s|}} \left[ \psi(r) - \psi^{(0)} \frac{1}{r^{|m-\alpha s|}} \right], \end{aligned}$$

where  $\lambda_m$  is the self-adjoint extension parameter. In [65] is shown that there is a relation between the self-adjoint extension parameter  $\lambda_m$  and the number  $\theta$  above. The number  $\theta$  is associated with the mapping of deficiency subspaces and extend the domain of operator to make it self-adjoint. The self-adjoint extension parameter  $\lambda_m$  have a physical interpretation: it represents the scattering length [67] of  $H_{0,\lambda_m}$  [65]. For  $\lambda_m = 0$  we have the free Hamiltonian (without the  $\delta$  function) with regular wave functions at origin and for  $\lambda_m \neq 0$  the boundary condition in (22) allows a  $r^{-|m-\alpha s|}$  singularity in the wave functions at origin.

#### IV. SCATTERING AND BOUND STATE ANALYSIS

The general solution for Eq. (15) in the  $r \neq 0$  region can be written as

$$\psi_m(r) = a_m J_{|m-\alpha s|}(kr) + b_m Y_{|m-\alpha s|}(kr), \quad (23)$$

with  $a_m$  and  $b_m$  being constants and  $J_\nu(z)$  and  $Y_\nu(z)$  are the Bessel functions of first and second kind, respectively. Upon replacing  $\psi_m(r)$  in the boundary condition (22), one obtain

$$\lambda_m a_m \mathcal{A} k^{|m-\alpha s|} = b_m \left[ \mathcal{B} k^{-|m-\alpha s|} - \lambda_m \left( \mathcal{C} k^{|m-\alpha s|} + \mathcal{B} \mathcal{D} k^{-|m-\alpha s|} \lim_{r \rightarrow 0^+} r^{2-2|m-\alpha s|} \right) \right], \quad (24)$$

with

$$\begin{aligned} \mathcal{A} &= \frac{1}{2^{|m-\alpha s|} \Gamma(1 + |m - \alpha s|)}, & \mathcal{B} &= -\frac{2^{|m-\alpha s|} \Gamma(|m - \alpha s|)}{\pi}, \\ \mathcal{C} &= -\frac{\cos(\pi|m - \alpha s|) \Gamma(-|m - \alpha s|)}{\pi 2^{|m-\alpha s|}}, & \mathcal{D} &= \frac{k^2}{4(1 - |m - \alpha s|)}. \end{aligned} \quad (25)$$

In Eq. (24),  $\lim_{r \rightarrow 0^+} r^{2-2|m-\alpha s|}$  is divergent if  $|m - \alpha s| \geq 1$ , hence  $b_m$  must be zero. On the other hand,  $\lim_{r \rightarrow 0^+} r^{2-2|m-\alpha s|}$  is finite for  $|m - \alpha s| < 1$ . This means that there arises the contribution of the irregular solution  $Y_{|m-\alpha s|}(kr)$ . Here, the presence of an irregular solution contributing to the wave function stems from the fact the Hamiltonian  $H_0$  is not a self-adjoint operator when  $|m - \alpha s| < 1$  (cf., Section III), hence such irregular solution must be associated with a self-adjoint extension of the operator  $H_0$  [68, 69]. Thus, for  $|m - \alpha s| < 1$ , we have

$$\lambda_m a_m \mathcal{A} k^{|m-\alpha s|} = b_m (\mathcal{B} k^{-|m-\alpha s|} - \lambda_m \mathcal{C} k^{|m-\alpha s|}), \quad (26)$$

and by substituting the values of  $\mathcal{A}$ ,  $\mathcal{B}$  and  $\mathcal{C}$  into above expression we find

$$b_m = -\mu_m^{\lambda_m}(k, \alpha) a_m, \quad (27)$$

where

$$\mu_m^{\lambda_m}(k, \alpha) = \frac{\lambda_m k^{2|m-\alpha s|} \Gamma(1 - |m - \alpha s|) \sin(\pi|m - \alpha s|)}{\lambda_m k^{2|m-\alpha s|} \Gamma(1 - |m - \alpha s|) \cos(\pi|m - \alpha s|) + 4^{|m-\alpha s|} \Gamma(1 + |m - \alpha s|)}. \quad (28)$$

Since a  $\delta$  function is a very short range potential, it follows that the asymptotic behavior of  $\psi_m(r)$  for  $r \rightarrow \infty$  is given by [70]

$$\psi_m(r) \sim \sqrt{\frac{2}{\pi k r}} \cos \left( kr - \frac{|m|\pi}{2} - \frac{\pi}{4} + \delta_m^{\lambda_m}(k, \alpha) \right), \quad (29)$$

where  $\delta_m^{\lambda_m}(k, \alpha)$  is a scattering phase shift. The phase shift is a measure of the argument difference to the asymptotic behavior of the solution  $J_{|m|}(kr)$  of the radial free equation which is regular at the origin. By using the asymptotic behavior of the Bessel functions [71] into Eq. (23) one obtain

$$\psi_m(r) \sim \sqrt{\frac{2}{\pi kr}} \left[ \cos \left( kr - \frac{\pi|m - \alpha s|}{2} - \frac{\pi}{4} \right) - \mu_m^{\lambda_m}(k, \alpha) \sin \left( kr - \frac{\pi|m - \alpha s|}{2} - \frac{\pi}{4} \right) \right]. \quad (30)$$

By comparing the above expression with Eq. (29), we have

$$\delta_m^{\lambda_m}(k, \alpha) = \Delta_m(\alpha) + \theta_m^{\lambda_m}(k, \alpha), \quad (31)$$

with

$$\Delta_m(\alpha) = \frac{\pi}{2}(|m| - |m - \alpha s|), \quad (32)$$

the phase shift of the AC scattering and

$$\theta_m^{\lambda_m}(k, \alpha) = \arctan [\mu_m^{\lambda_m}(k, \alpha)]. \quad (33)$$

Therefore, the scattering operator  $S_m^{\lambda_m}(k, \alpha)$  ( $S$ -matrix) for the self-adjoint extension is

$$S_m^{\lambda_m}(k, \alpha) = e^{2i\delta_m^{\lambda_m}(k, \alpha)} = \left[ \frac{1 + i\mu_m^{\lambda_m}(k, \alpha)}{1 - i\mu_m^{\lambda_m}(k, \alpha)} \right] e^{2i\Delta_m(\alpha)}. \quad (34)$$

Using Eq. (28), we have

$$S_m^{\lambda_m}(k, \alpha) = \left[ \frac{\lambda_m k^{2|m-\alpha s|} \Gamma(1 - |m - \alpha s|) e^{i\pi|m-\alpha s|} + 4^{|m-\alpha s|} \Gamma(1 + |m - \alpha s|)}{\lambda_m k^{2|m-\alpha s|} \Gamma(1 - |m - \alpha s|) e^{-i\pi|m-\alpha s|} + 4^{|m-\alpha s|} \Gamma(1 + |m - \alpha s|)} \right] e^{2i\Delta_m(\alpha)}. \quad (35)$$

Hence, for any value of the self-adjoint extension parameter  $\lambda_m$ , there is an additional scattering.

If  $\lambda_m = 0$ , we achieve the corresponding result for the AC problem with Dirichlet boundary condition; in this case, we recover the expression for the scattering matrix found in Ref. [72],

$S_m^0(k, \alpha) = e^{2i\Delta_m(\alpha)}$ . If we make  $\lambda_m = \infty$ , we get  $S_m^\infty(k, \alpha) = e^{2i\Delta_m(\alpha) + 2i\pi|m-\alpha s|}$ .

In accordance with the general theory of scattering, the poles of the  $S$ -matrix in the upper half of the complex plane [73] determine the positions of the bound states in the energy scale. These poles occur in the denominator of (35) with the replacement  $k \rightarrow i\kappa$ , with  $\kappa^2 = -2ME$ ,  $E < 0$ . Thus,

$$\lambda_m (i\kappa)^{2|m-\alpha s|} \Gamma(1 - |m - \alpha s|) e^{-i\pi|m-\alpha s|} + 4^{|m-\alpha s|} \Gamma(1 + |m - \alpha s|) = 0. \quad (36)$$

Solving the above equation for  $E$ , we found the bound state energy

$$E = -\frac{2}{M} \left[ -\frac{1}{\lambda_m} \frac{\Gamma(1 + |m - \alpha s|)}{\Gamma(1 - |m - \alpha s|)} \right]^{1/|m-\alpha s|}, \quad (37)$$



for  $\lambda_m < 0$ . Hence, the poles of the scattering matrix only occur for negative values of the self-adjoint extension parameter, when we have scattering and bound states. In this latter case, the scattering operator can be expressed in terms of the bound state energy

$$S_m^{\lambda_m}(k, \alpha) = e^{2i\Delta_m(\alpha)} \left[ \frac{e^{2i\pi|m-\alpha s|} - (\kappa/k)^{2|m-\alpha s|}}{1 - (\kappa/k)^{2|m-\alpha s|}} \right]. \quad (38)$$

In a previous work [74] (cf. also, [75–77] for analogous systems), using another self-adjoint extension approach, the energy bound state for the present system was determined in terms of the physics of the problem, and it read

$$E = -\frac{2}{Ma^2} \left[ \left( \frac{\alpha + |m - \alpha s|}{\alpha - |m - \alpha s|} \right) \frac{\Gamma(1 + |m - \alpha s|)}{\Gamma(1 - |m - \alpha s|)} \right]^{1/|m - \alpha s|}, \quad (39)$$

where  $a$  is a very small radius smaller than the Compton wavelength  $\lambda_C$  of the electron [78], which comes from the regularization of the  $\delta$  function. By comparing Eq. (37) with the Eq. (39) we have

$$\frac{1}{\lambda_m} = -\frac{1}{a^{2|m-\alpha s|}} \left( \frac{\alpha + |m - \alpha s|}{\alpha - |m - \alpha s|} \right). \quad (40)$$

The above relation is only valid for  $\lambda_m < 0$ , consequently we have  $|\alpha| \geq |m - \alpha s|$  and due to  $|m - \alpha s| < 1$  it is sufficient to consider  $|\alpha| \geq 1$  to guarantee  $\lambda_m$  to be negative. A necessary condition for a  $\delta$  function generates an attractive potential, which is able to support bound states, is that the coupling constant must be negative. Thus, the existence of bound states requires

$$\alpha \leq -1. \quad (41)$$

Also, it seems from the above equation and from (13) that we must have  $gV\lambda < 0$  and there is a minimum value for this product to ensure the presence of a bound state.

The scattering amplitude  $f(k, \alpha)$  can be now obtained using the standard methods of scattering theory, namely

$$\begin{aligned} f(k, \alpha) &= \frac{1}{\sqrt{2\pi ik}} \sum_{m=-\infty}^{\infty} \left( e^{2i\delta_m^{\lambda_m}(k, \alpha)} - 1 \right) e^{im\varphi} \\ &= \frac{1}{\sqrt{2\pi ik}} \left\{ \sum_{|m-\alpha s| \geq 1} (e^{2i\Delta_m(\alpha)} - 1) e^{im\varphi} \right. \\ &\quad \left. + \sum_{|m-\alpha s| < 1} (e^{2i\Delta_m(\alpha)} \left[ \frac{1 + i\mu_m^{\lambda_m}(k, \alpha)}{1 - i\mu_m^{\lambda_m}(k, \alpha)} \right] - 1) e^{im\varphi} \right\}. \end{aligned} \quad (42)$$

The first sum is the AC amplitude (i.e., in the absence of the  $\delta$  function), while the second sum is the contribution that come from the singular solutions. In the above equation we can see that

the scattering amplitude is energy dependent (cf., Eq. (28)). This is a clearly manifestation of the known non-conservation of the helicity in the AC scattering [79], because the only length scale in the nonrelativistic problem is set by  $1/k$ , so it follows that the scattering amplitude would be a function of the angle alone, multiplied by  $1/k$  [80]. In fact, the failure of helicity conservation expressed in Eq. (42), it stems from the fact that the  $\delta$  function singularity make the Hamiltonian and the helicity nonself-adjoint operators [81–84]. By expressing the helicity operator,  $\hat{h} = \mathbf{\Sigma} \cdot \mathbf{\Pi}$ , in terms of the variables used in (14), we attain

$$\hat{h} = \begin{pmatrix} 0 & -i \left( \partial_r + \frac{|m - \alpha s| + 1}{r} \right) \\ -i \left( \partial_r - \frac{|m - \alpha s|}{r} \right) & 0 \end{pmatrix}. \quad (43)$$

Notice under a parity  $\pi$  transformation  $\hat{h} \rightarrow \pi^\dagger \hat{h} \pi = -\hat{h}$ , that comes immediately from the parity transformation  $\pi^\dagger r \pi = -r$ . This is in fact the helicity odd-parity property. The helicity operator share the same issue as the Hamiltonian operator in the interval  $|m + \alpha s| < 1$ , i.e., it is not self-adjoint [85, 86]. Despite that on a finite interval  $[0, L]$ ,  $\hat{h}$  is a self-adjoint operator with domain in the functions satisfying  $\xi(L) = e^{i\theta} \xi(0)$ , it does not admit a self-adjoint extension on the interval  $[0, \infty)$  [87], and consequently it can be not conserved, thus the helicity conservation is broken due to the presence of the singularity at the origin [80, 82].

## V. CONCLUSION

We have studied the spin-1/2 AC scattering problem with a Lorentz-violating and CPT-odd nonminimal coupling between fermions and the gauge field in the context of the Dirac equation. It has been shown that there is an additional scattering for any value of the self-adjoint extension parameter and for negative values of this parameter there is non-zero energy bound states. The scattering amplitude show a energy dependency, so the helicity in not conserved. This stem from the fact that the helicity operator is not a self-adjoint extension operator. Therefore, it does not represent a quantum observable and does not correspond to a conserved quantity.

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